First midterm test, 27^{th} October 2023

Name:

Points: /25

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1. Show that
$$1 + 3 + \ldots + (2n - 1) = n^2$$
holds for every natural number $n \in \mathbb{N}$. *(Note that the left hand side can be written as $\sum_{i=1}^{n}(2i - 1)$.)*

Points: /4

2. Let $u = (1, -1, 2, 4)$, $v = (2, 1, 0, 1)$, $w = (0, 1, 0, 3)$ and $z = (1, -1, 1, -2)$. Compute $u + 2v$ and decide, whether this vector belongs to the span of $\{w, z\}$.

Points: /5

3. Compute the rank of
$$\begin{pmatrix} -1 & 2 & 9 & 5 \\ 2 & 2 & 3 & -1 \\ -1 & 0 & 2 & 2 \\ 0 & 2 & 7 & 3 \end{pmatrix}$$

Points: /4

4. Decide, whether
$$C = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$
is regular or not. If it is, compute C^{-1}.

Points: /6

5. Consider the quadratic form
$$Q(x, y, z) = x^2 + y^2 - xy + z^2.$$
Is there $(x, y, z) \in \mathbb{R}^3$ such that $Q(x, y, z)$ is negative? If yes, find it, if not, justify your answer.

Points: /6