

Name: _____

Points: /25

1. Show that

$$2 + 5 + \dots + (3n - 1) = \frac{n(3n + 1)}{2}$$

holds for every natural number $n \in \mathbb{N}$. (Note that the left hand side can be written as $\sum_{i=1}^n (3i - 1)$.)

Points: /6

2. Compute the rank of

$$\begin{pmatrix} 2 & 2 & 3 \\ -1 & 0 & 2 \\ 0 & 2 & 7 \end{pmatrix}$$

Points: /3

3. Find all solutions to

$$\begin{aligned} 2x + y - z + 3t &= 3 \\ -x + 2y + 2z - t &= 1 \\ 3x - y - 3z + 4t &= 2. \end{aligned}$$

Points: /6

4. Compute

$$\det \begin{pmatrix} 1 & -1 & 0 & 3 \\ 0 & 2 & 2 & 1 \\ -1 & 1 & 1 & 1 \\ 0 & 0 & 2 & -1 \end{pmatrix}$$

Points: /4

5. Let

$$C = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Is C regular? If yes, compute C^{-1} .

Points: /6