First midterm test, 10^{th} March 2023

Name:

Points: /25

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1. Let $u = (2,1,0)$ and $v = (-1,1,1)$. Decide, whether

$$u + 2v \in \operatorname{span}\{(1,1,1),\ (-1,2,1)\}$$

and justify your claim.

Points: /4

2. Compute a matrix X given as

$$X = \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \end{pmatrix} \begin{pmatrix} 2 & 2 \end{pmatrix}.$$

Points: /4

3. Find the characteristic polynomial of

$$\begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

and verify that $\lambda_1 = 0$, $\lambda_2 = 2$ and $\lambda_3 = 3$ are the eigenvalues of the matrix. Then find the eigenvector which corresponds to $\lambda_2 = 2$.

Points: /6

4. Find all solutions to

$$\begin{aligned} 2x - 3y + z &= 5 \\ x + y + z &= 0 \\ x + 2y - 3z &= -1. \end{aligned}$$

Points: /5

5. Compute the determinant of

$$\begin{pmatrix} 1 & 0 & -2 & 1 \\ 0 & 2 & 1 & 1 \\ -2 & 1 & 1 & 0 \\ 3 & 1 & 1 & 1 \end{pmatrix}.$$

Points: /6