First midterm test, 10^{th} March 2023

Name:

Points: /25

1. Do the vectors $(1, -1, 1)$, $(-1, 1, 1)$ and $(1, 1, -1)$ form a basis in $\mathbb{R}^3$? Justify your claim.

Points: /4

2. Compute a matrix X given as
$$X = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}.$$

Points: /5

3. Find the characteristic polynomial of
$$\begin{pmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$$
and verify that $\lambda_1 = 1$, $\lambda_2 = 3$ and $\lambda_3 = 4$ are the eigenvalues of the matrix. Then find the eigenvector which corresponds to $\lambda_1 = 1$.

Points: /6

4. Find all solutions to
$$\begin{aligned} 2x - 3y + z &= 5 \\ x + y + z &= 0 \\ x + 2y - 3z &= -1. \end{aligned}$$

Points: /5

5. Find the symmetric matrix A corresponding to the quadratic form
$$Q(x, y, z) = 2x^2 + 2y^2 + z^2 + xy + 2yz + 2xz^2$$
and decide about its definiteness. Justify your claim.

Points: /5