First midterm test, 10^{th} March 2023

Name:

Points: /25

---

1. Let

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & 2 & 0 \\ 1 & 2 & 0 \\ 0 & 1 & -1 \end{pmatrix}.$$

   Compute, if possible

   (a) $A + B$

   (b) AB

   (c) BA

Points: /3

2. Is $C = \begin{pmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{pmatrix}$ a regular matrix? If yes, compute C^{-1}.

Points: /6

3. Find all eigenvalues of $D = \begin{pmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$.

Points: /5

4. Find all solutions to

$$\begin{aligned} 2x - y + z - 3t &= 4 \\ 2x + y - z + t &= 1 \\ 3x - 2y + 2z - 3t &= 2 \\ 5x + y - z + 2t &= -1. \end{aligned}$$

Points: /5

5. Do the vectors $v_1 = (1, 1, 1)$, $v_2 = (1, 2, 1)$, $v_3 = (0, 0, 1)$ form a basis of $\mathbb{R}^3$? If yes, find coordinates of $v = (1, 0, 4)$ with respect to this basis.

Points: /6