

Name: _____

Points: /25

1. Prove, that

$$1 + 3 + 3^2 + \dots + 3^n = \frac{3^{n+1} - 1}{2}$$

holds for all $n \in \mathbb{N}$.

Points: /4

2. Let

$$A = \begin{pmatrix} 0 & 3 & -1 \\ 1 & 0 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 2 & 1 \\ 1 & 2 & 2 \end{pmatrix}.$$

Compute, if possible

- (a) $A + B$
- (b) AB
- (c) BA^T

Points: /3

3. Is $C = \begin{pmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{pmatrix}$ a regular matrix? If yes, compute C^{-1} .

Points: /6

4. Find all eigenvalues of $D = \begin{pmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$.

Points: /6

5. Do the vectors $v_1 = (1, 0, 1)$, $v_2 = (-1, 1, 0)$, $v_3 = (0, 3, 1)$ form a basis of \mathbb{R}^3 ? If yes, find coordinates of $v = (0, 0, 4)$ with respect to this basis.

Points: /6