

Second midterm test – sample, 31th March 2023

Name: _____

Points: /25

1. Let $\{a_n\}_{n=1}^{\infty}$ is a sequence given recursively as

$$a_1 = 1, \quad a_{n+1} = a_n - \frac{2n+1}{n^2(n+1)^2}.$$

Find an explicit formula for a_n and justify your claim.

Points: /4

2. Compute

$$\lim \sqrt{n^2 + 4n} - n$$

Points: /5

3. Let

$$f(x, y) = (x - 3 + y)^2 + 2y.$$

Write a function $g(t) = f(2t, 1 - t)$ and sketch its graph.

Points: /4

4. Examine

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy\sqrt{x}}{\sqrt{x^2 + y^2}}.$$

Points: /6

5. Write the second order Taylor polynomial of

$$f(x, y) = (x + 2y)e^y$$

at point $(x_0, y_0) = (1, 0)$.

Points: /6