Third midterm test, 18^{th} December 2023

Name:

Points: /25

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1. Write the third-degree Taylor polynomial at $x_0 = 0$ of

$$f(x) = (1 + x)e^x.$$

Points: /5

2. Find the maximal domain of

$$f(x, y) = \frac{x^2 + y}{1 - \sqrt{x^2 + y}}$$

and make its sketch.

Points: /5

3. Sketch the set

$$M = \{(x, y) \in \mathbb{R}^2,\ 2x + y^2 > 2,\ x > y\}$$

and decide, whether it is open or closed. Justify your claim.

Points: /5

4. Let

$$f(x, y) = \frac{x^2 + 2y}{y^2 + 1}$$

Compute ∇f and $\nabla^2 f$.

Points: /5

5. Write the equation of the tangent plane to the function

$$f(x, y) = y\sqrt{x^2 + y}$$

at $(x_0, y_0) = (2, 5)$.

Points: /5

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1. Compute
$$\lim_{x\to 0} \frac{x \sin x}{1-\cos x}.$$

Points: /5

2. Sketch the set
$$M = \{(x,y) \in \mathbb{R}^2,\ x^2 + 4y^2 < 16,\ x < y\}$$
and decide, whether it is open or closed. Justify your claim.

Points: /5

3. Examine the limit
$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}.$$

Points: /5

4. Let
$$f(x,y) = \frac{x+y^2}{y^2+1}$$
Compute ∇f and $\nabla^2 f$.

Points: /5

5. Write the second-degree Taylor polynomial centered at $(0,0)$ of
$$f(x,y) = (y+1)e^x.$$

Points: /5

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1. Compute
$$\lim_{x\to 0} \frac{e^x - x - 1}{x^2 \cos x}.$$

Points: /5

2. Write the third-degree Taylor polynomial at $x_0 = 0$ of
$$f(x) = (1 + x)e^x.$$

Points: /5

3. Find the countour lines at heights $z_0 = -1,\ 0,\ 1$ of
$$f(x, y) = (x + y)^2 - 1$$
and make their sketch.

Points: /5

4. Let
$$f(x, y) = \frac{x + y^2}{x^2 + 1}$$
Compute ∇f and $\nabla^2 f$.

Points: /5

5. Write the equation of the tangent plane to the function
$$f(x, y) = x\sqrt{x^2 + y}$$
at $(x_0, y_0) = (2, 5)$.

Points: /5