

Third midterm test – sample, 18th December 2023

Name: _____

Points: /25

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1. Use the l'Hospital rule to compute

$$\lim_{x \rightarrow 0} x^2 \cot x \frac{1}{\arctan x}$$

Points: /5

2. Sketch the set

$$\{(x, y) \in \mathbb{R}^2, x^2 \leq 3, y^2 \leq x\}$$

and determine, whether it is closed or open. Justify your claim.

Points: /5

3. Find and sketch the domain and the contour lines at heights $z_0 = -1, 0, 1$ of

$$f(x, y) = \sqrt{y^2 + 2x} - 2$$

Points: /5

4. Compute ∇f and $\nabla^2 f$ of

$$f(x, y) = \frac{x^2 + y^2}{e^x}$$

Points: /5

5. Write the second order Taylor polynomial of

$$f(x, y) = \frac{x}{x^2 + y}$$

centered at point $(1, 0)$.

Points: /5