Third midterm test – sample no. 2, 9^{th} May 2023

Name:

Points: /25

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1. Does the equation
$$x^2y - xy^2 = 0$$
define a function $y(x)$ on the neighborhood of $(1, 1)$? If yes, compute $y'(1)$.

Points: /5

2. Examine the local extremes of
$$f(x, y) = x^3 + x^2y - y^2 - 4y$$

Points: /6

3. Rewrite the second order equation
$$2y'' + 3y' + 6y = 0$$
into a system of first-order differential equations.

Points: /4

4. The fundamental system of
$$x' = Ax$$
is
$$\left\{ e^t \begin{pmatrix} 3 \\ 1 \end{pmatrix}, e^{-2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$$
Find the particular solution satisfying
$$x(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Points: /4

5. Find the critical points of the system
$$x'(t) = x(y - 1)$$
$$y'(t) = (y - 1)y$$
Then sketch several representative trajectories in a phase plane.

Points: /6