Exam test – 22nd May 2023

Name:

Points: /100

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1. Solve

$$\begin{aligned} x + y + z + t &= 4 \\ 2x + z &= 5 \\ -z + 3t &= 2 \\ x - y - z + 2t &= 3. \end{aligned}$$

Points: /20

2. Consider the function

$$f(x, y) = \frac{x}{\sqrt{x^2 + y^2}}.$$

   (a) Describe and sketch the maximal domain of f.

   (b) Find and sketch the contour lines at heights $c = -2, -1, 0, 1, 2$.

   (c) Compute all partial derivatives of the first order.

   (d) Compute all partial derivatives of the second order.

Points: /25

3. Let a function $f(x, y) = 2x^2 + 2xy + y^2$ be defined on a set

$$M = \{(x, y) \in \mathbb{R}^2,\ x^2 + y^2 \leq 25,\ x - y \geq 0\}.$$

   (a) Sketch the set M.

   (b) Find the points where the maximum and minimum are attached. Evaluate the function at these points.

Points: /30

4. Consider the difference equation

$$y(n + 2) + 8y(n + 1) - 20y(n) = 48 \cdot 2^n$$

   (a) Find all solutions to the appropriate homogeneous equation.

   (b) Find one solution to the given equation.

   (c) Based on the previous steps, write all solutions to the given equation.

   (d) Find the one particular solution satisfying

$$y(0) = 6, \text{ and } y(1) = 4.$$

Points: /25