Exam test 5th January 2024

Name:

Points: /100

1. Let A and B be given as
$$A = \begin{pmatrix} 2 & 2 & -1 \\ 1 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ 0 & 1 \end{pmatrix}.$$
   - Compute $C = BA$.
   - Compute C^{-1}.

Points: /25

2. Consider a function $f(x) = \sqrt{x^2 + 6x + 3} - \sqrt[3]{x+1}$.
   - Find a maximal domain of f (i.e., find all $x \in \mathbb{R}$ for which is $f(x)$ well defined.)
   - Determine, whether the function is even, odd or none of these. Justify your claim.
   - Compute $f'(x)$ and $f''(x)$.

Points: /20

3. Examine the course of the function
$$f(x) = 2x^2 e^{-x}.$$
(Recall that the following six steps are needed: 1, determine the domain, 2, examine parity, intersections with axis, etc., 3, examine the behavior of the function on the edges of the domain (including asymptotes), 4, examine the monotonicity of the function (including local maxima/minima), 5, examine convexity/concavity (including points of inflexion), 6, draw a sketch of a graph)

Points: /30

4. Let f be given as
$$f(x, y) = \sqrt{x^2 + 4}\, e^{xy}.$$
   - Find ∇f and $\nabla^2 f$.
   - Write the second degree Taylor polynomial centered at the point $(0, 0)$.
   - Use the result of the previous point to compute the approximate value of $\sqrt{5}\, e^{0.1}$.

Points: /25