Exam test 12th January 2024

Name:

Points: /100

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1. Find all eigenvalues and appropriate eigenvectors of the matrix A given as

$$A = \begin{pmatrix} 1 & 0 & 12 \\ 2 & -5 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

Points: /25

2. Examine the course of the function

$$f(x) = x^2(4 - x^2).$$

(Recall that the following six steps are needed: 1, determine the domain, 2, examine parity, intersections with axis, etc., 3, examine the behavior of the function on the edges of the domain (including asymptotes), 4, examine the monotonicity of the function (including local maxima/minima), 5, examine convexity/concavity (including points of inflexion), 6, draw a sketch of a graph)

Points: /30

3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given as

$$f(x, y) = \frac{x^2 + 2}{y}.$$

- Find and sketch the maximal domain of f.
- Decide whether is the domain from the preivous step open or closed. Justify your claim.
- Find and sketch the contour lines at heights $c = -2, -1, 0, 1, 0$.

Points: /20

4. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given as

$$f(x, y) = y^3 + x^2 - 6xy + 3x + 6y - 7.$$

- Find ∇f and use it do determine the stationary points.
- Compute $\nabla^2 f$.
- Classify the local extrema appearing in the stationary points.

Points: /25