Exam test – 7th June 2023

Name:

Points: /100

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1. The sequence a_n is given as

$$a_n = \frac{2k+1}{(k+1)^2k^2}$$

- Find $\lim a_n$.
- Denote by $s_k = \sum_{n=1}^{k} a_k$ the $k-th$ partial sum. Write s_1, s_2, s_3 and s_4.
- Decide, whether

$$\sum_{n=1}^{\infty} a_n$$

is finite or not. Justify your claim.

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2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function given as

$$f(x,y) = \frac{xy}{x^2 - xy}$$

- Find (and sketch) the maximal domain of f.
- Compute

$$\lim_{(x,y)\to(0,0)} f(x,y).$$

- Compute ∇f.
- Compute $\nabla^2 f$.

Points: /25

3. Let $(x_0, y_0) = (2, 0)$. Is there a function $y(x)$ uniquely determined by

$$x^2y - xe^y = 2$$

on a neighborhood of (x_0, y_0)? Justify your claim. If there is such function then

- decide, whether is this function increasing or decreasing in the point x_0.
- Write the second order Taylor polynomial for $y(x)$ centered in x_0.

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4. Consider

$$y(n+2) - 6y(n+1) + 5y(n) = n + 1.$$

- Find all solutions to the appropriate homogeneous system.
- Find one particular solution to the above system.
- Based on the previous steps, write all solutions to the above system.
- Write the particular solution satisfying $y(0) = 3$ and $y(1) = 1$.

Points: /30