**Exam test 19th January 2024**

Name:

Points: /100

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1. Let there be given vectors $u = (2, 1, 1)$, $v = (1, -1, 1)$, $w = (1, 1, 1)$, and $x = (-1, 0, 2)$.
   - Decide, whether the vectors $u$, $v$ and $w$ forms a basis of $\mathbb{R}^3$.
   - Write the coordinates of $x$ with respect to the basis $u, v, w$.

   Points: /30

2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given as
$$f(x, y) = \log(x^2 + y + 1).$$
   - Determine and sketch the maximal domain of $f$.
   - Find and sketch the contour lines at heights $c = 0, -1, 1$.
   - Compute $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

   Points: /25

3. Compute the first and the second gradient of
$$f(x, y) = e^{x^2+y}\sqrt{1 + y^2}.$$

   Points: /20

4. Examine the local extrema of
$$f(x, y) = x^2y^2 - x^2 - y^2.$$

   Points: /25