Exam test – 19th June 2023

Name:

Points: /100

1. Consider vectors
$$v_1 = (1,0,2),\ v_2 = (0,1,1),\ v_3 = (1,-1,2),\ w = (2,2,0)$$
   - Do the vectors v_1, v_2, v_3 form a basis of $\mathbb{R}^3$? Justify your claim.
   - Write w as a linear combination of v_1, v_2 and v_3 (find the coordinates of w with respect to the basis $\{v_1, v_2, v_3\}$).

Points: /22

2. Let a_n be a sequence given as
$$a_n = \sqrt{n^2+1} - \sqrt{n^2+2n+2}$$
   - Compute $\lim a_n$.
   - Let $s_k = \sum_{n=1}^{k} a_n$ be the $k-$th partial sum. Write s_1, s_2, s_3
   - Does the sum
$$\sum_{n=1}^{\infty} a_n$$
     converge or diverge? Justify your claim.

Points: /25

3. Consider the equation
$$x^2 + y\cos x = 1.$$
   - Does this equation uniquely determine a function $y(x)$ on the neighborhood of the point $(0,1)$?
   - Compute $y'(0)$ and $y''(0)$.

Points: /23

4. Let $M \subset \mathbb{R}^2$ be given as
$$M := \{(x,y) \in \mathbb{R}^2,\ xy \geq \frac{1}{2},\ 0 < x \leq 2,\ 0 < y \leq 2\}$$
   and $f : M \to \mathbb{R}$ be given as
$$f(x,y) = x^2 + 4y^2.$$
   - Is M open or closed? Justify your claim.
   - Sketch M and dismantle it into the interior and boundary.
   - Find all points where there could be an extreme of f on M.
   - Determine the maximum and minimum of f on M.

Points: /30