Exam test – 23th June 2023

Name:

Points: /100

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1. Find the inverse matrix to

$$A = \begin{pmatrix} 2 & 0 \\ 2 & 1 \end{pmatrix}.$$

Then use it to find the matrix X which solves

$$AX = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} + \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \end{pmatrix}.$$

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2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given as

$$f(x, y) = 4x^3 - 3xy^2 + 12y.$$

- Find all stationary points of f.
- Decide, whether there are (local) extremes in the stationary points of f.

Points: /25

3. Consider the system

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}$$

- Find all solutions to the appropriate homogeneous system.
- Find one particular solution to the system with non-zero right hand side.
- Find the particular solution satisfying

$$\begin{pmatrix} x(0) \\ y(0) \\ z(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}.$$

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4. Consider the system

$$\begin{aligned} x' &= y^2 \\ y' &= xy. \end{aligned}$$

- Find all the critical points of the above system.
- Write the equation for trajectories (in particular, the equation for $\frac{dy}{dx}$).
- Solve the equation from the previous step and sketch the trajectories of the system.

Points: /25