Exam test 30th January 2024

Name:

Points: /100

1. Let A be given as
$$A = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & -2 \\ -4 & 0 & -1 \end{pmatrix}.$$
Find all eigenvalues and the appropriate eigenvectors of A.

Points: /25

2. Examine the course of the function
$$f(x) = \frac{x^2}{x-1}$$
(Recall that the following six steps are needed: 1, determine the domain, 2, examine parity, intersections with axis, etc., 3, examine the behavior of the function on the edges of the domain (including asymptotes), 4, examine the monotonicity of the function (including local maxima/minima), 5, examine convexity/concavity (including points of inflexion), 6, draw a sketch of a graph)

Points: /30

3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given as
$$f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$$
   - Determine and sketch the maximal domain of f.
   - Examine
$$\lim_{(x,y)\to(0,0)} f(x, y).$$
   - Compute ∇f.

Points: /23

4. Let $f : \mathbb{R} \to \mathbb{R}$ be given as
$$f(x, y) = (x^2 + 1)e^y - 2x$$
   - Evaluate ∇f at the point $(x_0, y_0) = (1, 0)$
   - Check, whether $f(1, 0) = 0$.
   - Based on the previous step, determine whether there is a uniquely determined function $y(x)$ on the neighborhood of the point $(1, 0)$ given by the equation $f(x, y) = 0$.
   - If there is such function, compute $y'(1)$.

Points: /22