Exam test – 14th August 2023

Name:

Points: /100

1. Let there are given vectors
$$u = (1, 0, -1, -1),$$
$$v = (2, 1, 1, -1),$$
$$w = (0, 3, -2, 2),$$
$$z = (1, 5, 3, 3).$$
   - Decide, whether u, v and w are linearly independent.
   - Express z as a linear combination of u, v and w if it is possible.

Points: /25

2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given as
$$f(x, y) = \frac{\sqrt{1 - y^2}}{x}$$
   - Describe and sketch the domain of f.
   - Find the contour lines at heights $c = -1$, $c = 0$ and $c = 1$ and sketch them.
   - Compute ∇f.
   - Compute $\nabla^2 f$.

Points: /25

3. Let the function f be given as
$$f(x, y) = x - y$$
and the set M be given as
$$\{(x, y) \in \mathbb{R}^2,\ x^2 + 2y^2 - 2xy \leq 25\}$$
   - Decompose M into interior and boundary.
   - Find stationary points of f in interior.
   - Use the Lagrange multiplier method to find the stationary points of f on the boundary.
   - Determine the maximum and minimum of f achieved on M.

Points: /30

4. The linear system of two unknowns
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & 4 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 11 \\ 1 \end{pmatrix}$$
has only one stationary point.
   - Find the stationary point.
   - Classify it (decide, whether it is stable/unstable node, stable/unstable spiral, saddle or center).

Points: /20