Name:

Points: /100

1. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given as
$$f(x, y) = xe^{x+y^2}.$$

   - Determine the domain of f.
   - Compute ∇f.
   - Compute $\nabla^2 f$.

Points: /20

2. Does the equation
$$x^2 y + e^{xy} - \sin y = 1$$

   determine a function $y(x)$ on a vicinity of the point $(0,0)$? If yes then

   - Compute $y'(0)$.
   - Compute $y''(0)$.
   - Write the equation of the tangent line to $y(x)$ at $x = 0$.

Points: /25

3. Examine the local extremes of
$$f(x, y) = xye^{x - \frac{1}{2}y^2}.$$

Points: /25

4. Consider the following (nonlinear) system of equations:

$$x' = y(y - x)$$
$$y' = x(y - x).$$

   - Determine all critical points of the system.
   - Find the equation for trajectories, i.e. an equation for $\frac{dy}{dx}$.
   - Solve the equation from the second step.
   - Sketch few trajectories into a phase plane (with arrows included).

Points: /30