Exam test – sample no. 1

Name:

Points: /100

1. Consider a matrix
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -3 & 3 \\ 1 & 1 & -1 \end{pmatrix}$$
   (a) Explain what is a singular matrix and what is a regular matrix.
   (b) Compute $\det A$.
   (c) Determine, whether A is singular or regular.
   (d) Find all vectors $v = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ fulfilling
$$Av = 0.$$

Points: /25

2. Consider an equation
$$x^3 + y^3 - 3xy - 3 = 0.$$
   (a) Does there exist a function $y(x)$ given by the equation on some neighborhood of a point $(1,2)$? Carefully verify all needed assumptions.
   (b) Compute $y'(1)$ for the function from the previous step.
   (c) Write an equation of the tangent line to the graph of the function y at the point $(1,2)$.

Points: /25

3. Consider the function
$$f(x,y) = x^2 - y^2$$
and a triangle M with vertices $(-1,1)$, $(-1,4)$, and $(3,0)$.
   (a) Sketch M and find the equations of the edges of the triangle.
   (b) Find the stationary points of f lying inside the triangle.
   (c) Find the points where there might be an extreme of f on the boundary of the triangle.
   (d) Determine the maximum and the minimum of f with respect to M and write the points where the maximium and minimum are achieved.

Points: /25

4. Consider a system of ODE
$$x'(t) = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 2 & 1 & 2 \end{pmatrix} x(t).$$
   (a) Find all solutions to the given system.
   (b) Find a solution which satisfies $x(0) = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$.

Points: /25