Exam test sample, winter semester 2023/2024

Name:

Points: /100

1. Consider the matrix
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -3 & 3 \\ 1 & 1 & -1 \end{pmatrix}.$$
   (a) Explain, what is a singular and what is a regular matrix.
   (b) Compute $\det A$.
   (c) Determine whether A is singular or regular.
   (d) Find all vectors $v = (x, y, z)$ such that
$$Av^T = 0.$$

Points: /25

2. Examine the course of the function
$$f(x) = \frac{x^2 - 3x}{x + 1}$$
   (Recall that the following six steps are needed: 1, determine the domain, 2, examine parity, intersections with axis, etc., 3, examine the behavior of the function on the edges of the domain (including asymptotes), 4, examine the monotonicity of the function (including local maxima/minima), 5, examine convexity/concavity (including points of inflexion), 6, draw a sketch of a graph)

Points: /30

3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given as
$$f(x, y) = \frac{x}{\sqrt{x^2 + y^2}}.$$
   - Determine the domain of f.
   - Compute ∇f.
   - Compute $\nabla^2 f$.

Points: /25

4. Examine the local extrema of
$$f(x, y) = x^2 + y^2 + 2x + 4y - 1.$$

Points: /20