

1. Consider a sequence  $a_n = \frac{\sqrt{n}}{n+100}$ .

- Is this sequence monotone? Prove your claim.
- Is this sequence bounded? Prove your claim.
- Compute  $\lim a_n$ .

2. Consider a function  $f = \frac{x^2}{x+1}$ .

- Find a set of all  $x \in \mathbb{R}$  for which  $f$  is well defined.
- Compute  $f'$  and  $f''$ .
- Consider a function  $f$  restricted to a domain  $[0, \infty)$ . Is such function one-to-one? Prove your claim.
- If the restriction from the previous answer is one-to-one, find an inverse  $f^{-1}$  to the restricted function.

3. Let the set  $M$  be given as

$$M = \{(x, y) \in \mathbb{R}^2, x^2 < y \text{ \& } y < x^2 + 6x + 9 \text{ \& } y < x^2 - 4x + 3\}$$

- Try to make a sketch of this set.
- Write an integral which can be used to compute the volume of  $M$ .
- Compute the volume of  $M$ .

4. Consider an equation

$$y'' + y' = \frac{1}{1 + e^x}$$

- Find all solution to the appropriate homogeneous problem.
- Use variation of constants to deduce one particular solution to the given problem.
- Write all solutions to the given problem.
- Find a particular solution fulfilling  $y(0) = 1, y'(0) = -1$ .