1. Use mathematical induction in order to show that

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

holds for all $n\in\mathbb{N}$.

Use this formula to compute

$$\lim_{n\to\infty}(-1)^n\left(\sum_{k=1}^{n}\frac{1}{k(k+1)}\right).$$

2. Use the Taylor polynomial to $f(x)=\sqrt{x}$ at point $x_0=9$ of degree $n=3$ to deduce the approximate value of $\sqrt{10}$.

3. Examine the course of function

$$f(x)=\frac{x^2-3x}{x+1}$$

(Recall that the following six steps are needed: 1, determine the domain, 2, examine parity, intersections with axis, etc., 3, examine the behavior of the function on the edges of the domain (including asymptotes), 4, examine the monotonicity of the function (including local maxima/minima), 5, examine convexity/concavity (including points of inflexion), 6, draw a sketch of a graph)

4. Consider an equation

$$y''+2y'+5y=\sin 2x.$$

   (a) Find all solution to the appropriate homogeneous problem.

   (b) Use a 'special right hand side' (undetermined coefficients) method to deduce one particular solution.

   (c) Write all solutions to the given problem.

   (d) Find a solution fulfilling $y(0)=2$, $y'(0)=-1$.