

1. Consider a sequence $a_n = \frac{((n-1)^2 - (n+1)^2) \cos(\pi n)}{n^2 + 2n}$.

(a) Decide, whether this sequence is increasing, decreasing or non-monotone. Justify your claim.

(b) Compute $\lim_{n \rightarrow \infty} a_n$. Explain all your steps.

2. Limits of function

(a) Formulate the l'Hospital rule.

(b) Use this rule to compute

$$\lim_{x \rightarrow 0} \frac{x \arctan x}{e^x + e^{-x} - 2}.$$

Do not forget to verify assumptions of the rule.

3. Antiderivatives:

(a) Write the 'integration-by-parts' formula.

(b) Describe, how to compute an integral by the method of substitution.

(c) Compute

$$\int x^5 e^{x^2} dx.$$

4. Consider an equation

$$y''' + y'' - y' - y = e^x$$

(a) Find all solution to the appropriate homogeneous problem.

(b) Use a 'special right hand side' method to deduce one particular solution.

(c) Write all solutions to the given problem.

(d) Find the particular solution fulfilling $y(0) = 0$, $y'(0) = 1$.