1. Find all solutions to the system

$$\begin{aligned} x + y + z + t &= 4 \\ 2x + z &= 1 \\ -z + 3t &= 7 \\ x - y - z + 2t &= 5 \end{aligned}$$

2. Let a function $f(x, y) = 2x^2 + 2xy + y^2$ be defined on a set

$$M = \{(x, y) \in \mathbb{R}^2,\ x^2 + y^2 \leq 25,\ x - y \geq 0\}.$$

   (a) Sketch the set M.

   (b) Is there a point where f attains maximum (resp. minimum) on M? Justify your answer.

   (c) Find the points where the maximum and minimum are attached. Evaluate the function at these points.

3. Consider a system of ODE

$$\mathbf{x}'(t) = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 2 & 1 & 2 \end{pmatrix} \mathbf{x}(t).$$

   (a) Find all solutions to the given system.

   (b) Find a solution which satisfies $\mathbf{x}(0) = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$.

4. Consider a vector field

$$F(x, y) = (2x^3y^4 + x, 2x^4y^3 + y).$$

   (a) Write a definition of a potential of a vector field.

   (b) Determine whether the given field F has a potential or not. Justify your answer.

   (c) If F has a potential, find it.