1 Extremes

1.1 Local extrema

1. Find and classify the local extrema of

$$f(x, y) = x^3 + x^2 y - 2y^3 + 6y.$$

2. Find and classify the local extrema of

$$f(x, y) = 4 + x^3 + y^3 - 3xy.$$

3. Find and classify the local extrema of

$$f(x, y) = x^2 + y^2 + 2x - 4y + 1.$$

4. Find and classify the local extrema of

$$f(x, y) = xye^{x - \frac{1}{2}y^2}.$$

1.2 Constraints

1. Find the maximum and the minimum of $f(x, y) = (x - 1)^2 + (y - 2)^2$ on a rectangle $[-1, 3] \times [0, 5]$ (i.e., $\{(x, y) \in \mathbb{R}^2, \ x \in [-1, 3], \ y \in [0, 5]\}$).

2. Find the maximum and the minimum value of $f(x, y) = 2x + 5y$ on the eclipse $9x^2 + 16y^2 = 144$.

3. Maximize $f(x, y, z) = x + z$ given $g(x, y, z) = x^2 + y^2 + z^2 = 1$.

4. Find the maximum and the minimum value of $f(x, y, z) = x$ under the constraints $x + 2z = 0$, $x^2 + y^2 + z^2 = 1$.

2 Integrals

2.1 Basic method

1. Compute

$$\int \left(x + \sqrt{\frac{1}{x}} \right) x^{-3/5} \, \mathrm{d}x$$

2. Compute

$$\int e^x + 2\sin x \, \mathrm{d}x$$

3. Compute

$$\int e^{4x} - 3e^{-x} + \cos x \, \mathrm{d}x$$

4. Compute

$$\int \frac{2x^2}{1 + x^2} \, \mathrm{d}x$$

5. Compute

$$\int \frac{x^3 + 4x + 2}{x^2 + 4} \, \mathrm{d}x$$

6. Compute

$$\int \frac{x^2}{1 + x} \, \mathrm{d}x$$