Make up test – 19th January 2024

1. Decide, whether $u = (1, 1, 0)$ is in the linear span of $v = (1, 0, 1)$ and $w = (0, 1, -1)$. Justify your claim.

2. Find the eigenvalues of
$$\begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix}$$

3. Compute
$$\lim_{x \to \infty} \frac{\log(1 + x^2)}{x \sin x}.$$

4. Find the maximal domain of
$$f(x) = \frac{1}{\sqrt{1 - \log x}}.$$

5. Compute f' and f'' for the function
$$f(x) = \sqrt{x}(x^2 + \cos x)$$

6. Find the stationary points of
$$f(x) = x^4 - 8x^3.$$

7. Find the first partial derivatives of
$$f(x, y) = (x^2 + xy)e^{2x}$$

8. Write the second degree Taylor polynomial of
$$f(x, y) = xe^{x-y}$$
centered at the point $(1, 1)$.