## 1 Samples

## Sample 1

- Compute

$$
\lim _{x \rightarrow 1+} \sqrt{x^{2}-1} \log (x-1)
$$

- Use the third-degree Taylor polynomial to approximate the value of $e^{-0.3}$
- Find and sketch the domain of

$$
f(x, y)=\frac{\sqrt{x}}{1-\sqrt{x^{2}-y^{2}}} .
$$

## Sample 2

- Compute

$$
\lim _{x \rightarrow \infty} x^{-3} e^{x}
$$

- Decide, whether

$$
A=\left\{(x, y) \in \mathbb{R}^{2}, x^{2}+2 x-y>0, x-y>1\right\}
$$

is closed or open and justify your answer. Sketch the set.

## Sample 3

- Write the fourth-degree polynomial at $x_{0}=0$ of $f(x)=$ $x^{2}\left(1+e^{x}\right)$.
- Find and sketch the boundary of

$$
A=\left\{(x, y) \in \mathbb{R}^{2}, x^{2} \geq 1, y<-1, x^{2}+y^{2}<16\right\} .
$$

- Sketch the countour lines of $f(x, y)=\log \left(x^{2}+4 y^{2}\right)$ at
- Let $f$ be given as

$$
f(x, y)=x^{2}+x y e^{2 y} .
$$

Compute $\nabla f$ and $\nabla^{2} f$.

- Write the second-degree Taylor polynomial for $f(x, y)=$ $x^{2} \sin y$ at the point $(2,0)$.
- Examine

$$
\lim _{(x, y) \rightarrow(0,0)} \frac{x y-y^{2}}{x^{2}+y^{2}} .
$$

- Let $f(x, y)=\frac{2 x-y}{x^{2}+y^{2}+1}$. Compute $\nabla f$ and $\nabla^{2} f$.
- Use the second-degree Taylor polynomial to find the approximate value of $\sqrt{3.2^{2}+3.9^{2}}$.
heights $z_{0}=-1,0,1$.
- Compute the derivative of $f(x, y)=x^{2}+2\left(x-y^{2}\right)$ at the point $(1,1)$ in direction $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.
- Write the equation of a plane tangent to a graph of $f(x, y)=x^{2}+\sqrt{5+y^{2}}$ at point $\left(x_{0}, y_{0}\right)=(1,2)$.

