1 2D gradients

Compute the gradients of the following functions

- $$a(x,y) = 2x^2y^3 - \frac{x}{y}$$
- $$b(x,y) = (x-y)\sin(x+y)$$
- $$c(x,y) = y\cos\left(x^2+y\right)$$
- $$d(x,y) = \frac{x^2+e^y}{y}$$
- $$e(x,y) = x\ln(x^2+y^2)$$
- $$f(x,y) = \frac{x^2+y}{x+y-1}$$
- $$g(x,y) = e^{x^2+2y}$$
- $$h(x,y) = x^2 + ye^{2x+1}$$
- $$i(x,y) = (x^2+1)^{y^2}$$
- $$j(x,y) = \frac{x}{y}\sqrt{x^2+y^2}$$
- $$k(x,y) = \sqrt{x^2+e^y}$$
- $$l(x,y) = \frac{x^2+1}{xe^y}$$

2 3D gradients

Compute the gradients of the following functions

- $$a(x,y,z) = 2x - y + z^2$$
- $$b(x,y,z) = x^2 + ye^{y+z}$$
- $$c(x,y,z) = (x+y^2)^5 + \frac{3}{z^2}$$
- $$d(x,y,z) = \left(\frac{x}{y}\right)^z$$
- $$e(x,y,z) = (x^2+z)\sin y$$
- $$f(x,y,z) = \frac{x+y\sin^2 z}{z}$$
- $$g(x,y,z) = x^2 + \sin^2(x(y+z^2))$$
- $$h(x,y,z) = \frac{xy}{z}$$
- $$i(x,y,z) = (x+y)\cos(z+y)$$

3 Chain rule

- Determine $\frac{\partial f}{\partial t}$ if
$$f(x,y) = \cos\left(yx^2\right), \quad x = t^4 - 2t, \ y = 1 - t^6$$
- Determine $\frac{\partial f}{\partial x}$ if
$$f(x,y) = x^2y^4 - 2y \quad \text{and} \quad y = \sin(x^2)$$
- Determine $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial p}$ if
$$f(x,y) = 4y\sin(2x), \quad x = 3u - p, \quad y = p^2u$$

4 Results

4.1 2D

$\nabla a = \left(4xy^3 - \frac{1}{y}, 6x^2y^2 + \frac{x}{y^2}\right)$, $\nabla b = (\sin(x+y) + (x-y)\cos(x+y), -\sin(x+y) + (x-y)\cos(x+y))$,
$\nabla c = \left(-2xy\sin\left(x^2+y\right), \cos\left(x^2+y\right) - y\sin\left(x^2+y\right)\right)$, $\nabla d = \left(\frac{2x}{y}, \frac{ye^y - (x^2+e^y)}{y^2}\right)$, $\nabla e = \left(\ln(x^2+y^2) + \frac{2x^2}{x^2+y^2}, \frac{2xy}{x^2+y^2}\right)$,
$\nabla f = \left(\frac{2x(x+y-1)-(x^2+y)}{(x+y-1)^2}, \frac{x-1-x^2}{(x+y-1)^2}\right)$, $\nabla g = \left(2xe^{x^2+2y}, 2e^{x^2+2y}\right)$, $\nabla h = \left(2x + 2ye^{2x+1}, e^{2x+1}\right)$,
$\nabla i = \left(2xy^2(x^2+1)^{y^2-1}, (x^2+1)^{y^2}\ln(x^2+1)\right)$, $\nabla j = \left(\frac{1}{y}\sqrt{x^2+y^2} + \frac{x}{y}\frac{x}{\sqrt{x^2+y^2}}, -\frac{x}{y^2}\sqrt{x^2+y^2} + \frac{x}{\sqrt{x^2+y^2}}\right)$,
$\nabla k = \left(\frac{x}{\sqrt{x^2+e^y}}, \frac{e^y}{2\sqrt{x^2+e^y}}\right)$, $\nabla l = \left(\frac{2x^2e^y - (x^2+1)e^y}{(xe^y)^2}, -\frac{x^2+1}{xe^y}\right)$

4.2 3D

$\nabla a = (2, -1, 2z)$, $\nabla b = (2x, (y+1)e^{y+z}, ye^{y+z})$, $\nabla c = \left(5(x+y^2)^4, 10y(x+y^2)^4, -\frac{3}{2}\frac{1}{z^3}\right)$,
$\nabla d = \left(\frac{z}{y}\left(\frac{x}{y}\right)^{z-1}, -\frac{zx}{y^2}\left(\frac{x}{y}\right)^{z-1}, \left(\frac{x}{y}\right)^{z}\ln\left(\frac{x}{y}\right)\right)$, $\nabla e = \left(2x\sin y, (x^2+z)\cos y, \sin y\right)$, $\nabla f = \left(\frac{x}{z}, \frac{\sin^2 z}{z}, \frac{2yz\sin z\cos z - x - y\sin^2 z}{z^2}\right)$,
$\nabla g = \left(2x + 2(y+z)^2\sin(x(y+z^2))\cos(x(y+z^2)), 2x\sin(x(y+z^2))\cos(x(y+z^2)), 4xz\sin(x(y+z^2))\cos(x(y+z^2))\right)$, $\nabla h = \left(\frac{y}{z}, \frac{x}{z}, -\frac{xy}{z^2}\right)$, $\nabla i = (\cos(y+z), \cos(y+z) - (x+y)\sin(z+y), -(x+y)\sin(z+y))$