

1. Decide, whether the vectors $u = (3, -1, -1, 0)$, $v = (1, 1, 0, 2)$ and $w = (0, -1, 0, 1)$ are linearly dependent or independent.

2. Find an intersection of the line p and the plane σ where p is given as

$$p: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + t \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, t \in \mathbb{R}$$

and σ consists of all points fulfilling the equation

$$x + 2y - z + 3 = 0.$$

3. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be given as

$$f(x, y) = x^2(x + \sin y).$$

Find ∇f .

4. Find all local maxima and minima of

$$f(x, y) = x^3 - 3xy + 3y^2.$$

5. Compute

$$\int_M xy^2 \, dx dy$$

where $M = \{(x, y) \in \mathbb{R}^2, x^2 \leq y \leq x, x \in [0, 1]\}$.

6. Find all solutions to a system

$$\mathbf{x}' = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \mathbf{x}.$$