

Solutions to samples of the 3rd midterm test

Sample a:

- 1, The function is not given, one of the conditions is not fulfilled.
- 2, Stationary points $(0, 0)$ and $(3, -9)$. We cannot decide about the first one, there is no extreme in the second one.
- 3, Minimum 6 is achieved in $(3, 0)$, maximum $67/4$ is achieved at $(\pm \frac{\sqrt{35}}{2}, -\frac{1}{2})$.
- 4, $\left\{ e^t \begin{pmatrix} 3 \\ 2 \end{pmatrix}, e^{-2} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$.
- 5, $2e^t \begin{pmatrix} 3 \\ 1 \end{pmatrix} - e^{-t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

Sample b:

- 1, Yes, it holds that $y'(1) = 1$.
- 2, No extreme at the stationary points $(0, 2)$, $(-4, 6)$ and $(1, 3/2)$.
- 3,

$$\begin{aligned}x' &= -3/2x - 3y \\ y' &= x\end{aligned}$$

4, $\frac{1}{3}e^t \begin{pmatrix} 3 \\ 1 \end{pmatrix} - \frac{4}{3}e^{-2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

- 5, Critical points are all points in the line $y = 1$ and then a point $(0, 0)$. The trajectories solves the system

$$\frac{\partial y}{\partial x} = \frac{y}{x}$$

(the sketch is not included, sorry for that).

Sample c:

- 1, Local minimum at $(2, -1)$
- 2, Maximum 1 is achieved at $(\pm 1, 0)$, minimum -1 is achieved at $(0, \pm 1)$
- 3,

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & -4 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t^2 + 1 \\ \sin t \end{pmatrix}$$

4, $\left\{ e^{-t} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1/3 \\ 2/3 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\}$

- 5, The critical points are all points in the line $x = 1$ and, moreover, the point $(0, 0)$. The trajectories are the solutions to the system

$$\frac{\partial y}{\partial x} = \frac{x}{y}$$

(the sketch is not included, sorry for that).