

1. Consider a matrix

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -3 & 3 \\ 1 & 1 & -1 \end{pmatrix}$$

(a) Explain what is a singular matrix and what is a regular matrix.

(b) Compute $\det A$.

(c) Determine, whether A is singular or regular.

(d) Find all vectors $\mathbf{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ fulfilling

$$A\mathbf{v} = 0.$$

2. Consider an equation

$$x^3 + y^3 - 3xy - 3 = 0.$$

(a) Does there exist a function $y(x)$ given by the equation on some neighborhood of a point $(1, 2)$? Carefully verify all needed assumptions.

(b) Compute $y'(1)$ for the function from the previous step.

(c) Write an equation of the tangent line to the graph of the function y at the point $(1, 2)$.

3. Find all solution to

$$\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} \mathbf{x}.$$

4. Compute

$$\int_{\mathcal{K}} xyz \, ds$$

where \mathcal{K} is a curve whose parametrization is

$$\begin{aligned} x &= t \\ y &= \frac{1}{3}\sqrt{8t^3} \\ z &= \frac{1}{2}t^2 \end{aligned}$$

where $t \in [0, 1]$.